Stabilizing bipedal walking on posts through multiple constraints

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Adaptability of the human locomotion system has been studied from the theoretical viewpoint of dynamical systems. The structure of a dynamical system consists of its time evolution rule, known simply as the dynamics, and its constraints, such as the initial states or boundary conditions that determine future convergent states. Initial state coordination by the system itself is the key to autonomous adaptive mechanisms. Exploring such mechanisms, our previous studies have focused on the variables encoding the attractor basins, called global variables. Global variables have been shown to enable the system to adapt to perturbations, by coordinating the initial states (constraints) of the system. Thus, initial state coordination by the global variables has been proposed as a mechanism for self-production of the constraints by a system. The adaptability of human locomotion extends to active integration into its locomotion of the motion of environmental objects such as walking on high-heeled shoes or on posts erected unstably. Dealing with bipedal walking on posts, this study expands the mechanism for self-production of the constraints. This study proposes a multiple structure for self-produced constraints and a framework for their description.

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I. INTRODUCTION

A feature of human locomotion is its adaptability to various conditions affecting it. This adaptability is not limited to passive avoidance of a perturbation such as a stumble [1], but also extends to active integration of environmental objects whose dynamics differs from that of the body. For example, when humans walk on high-heeled shoes or unstable posts [Fig. 1(a)], they are controlling the motion of environmental objects, feeling as if the objects are part of their body. How humans flexibly integrate environmental objects into intended movement is an important problem for biological cybernetics and the physiological and physical sciences that deal with embodiment phenomena [2,3]. This study is directed toward a theoretical understanding of the mechanisms for integrating environmental objects into the generation of locomotion. In particular, we investigate bipedal walking on unstable posts.

In general, a dynamical system consists of its time evolution rule, known simply as the dynamics, and its constraints, such as the initial states or boundary conditions that determine future convergent states. Modeling approaches using dynamical systems theory have conventionally described the human locomotion system by ordinary differential equations. In such systems, the only constraints are the initial states.

One of the most important theoretical problems for adaptive mechanisms in biological systems is to clarify how the system estimates its own state: estimates are described using the system's variables. In other words, how does the system know whether it is in a walking state or a falling state? In [4], we carefully investigated the basin structure of the system and found that the state of the whole system, that is, which basin of attraction the whole system lies in, is reflected in system variables referred to as *global variables*. In addition, it was shown that the global variables enable the system to move between the attractor basins by governing the initial state or the posture at the beginning of stance phase (BSP). As a result, we constructed a model that can adapt to strong external perturbations. The present paper focuses on such mechanisms, that is, "constraint production by the dynamics of system variables" or, more simply, *self-production of constraints*.

Until now, many studies modeling the adaptability of human walking have dealt with various types of perturbation to the walking system, such as a load caused by sudden collision or a physical impairment [5–11]. When affected by such perturbations, the solution of the system shifts out of the attractor basin of the walking state. Adaptability to these perturbations is accomplished by returning the solution to the attractor basin [9]. A mechanism for returning in a self-constraining manner has been proposed [4].

In this research, we model bipedal walking on unstable posts in a self-constraining manner. Theoretically, the system can maintain walking when a basin exists in the phase space and when the initial state is placed inside that basin. From a biomechanical viewpoint, the bipedal walking system moves forward by repetitions of the stance phase. An unstable post destabilizes the stance phase and may easily induce the system to fall [Fig. 1(b)]. As the number of posts increases, it becomes more difficult for the system to maintain walking; the basin becomes more difficult to find, and so it also becomes difficult to give an appropriate initial state.

In order to construct a model described by a dynamical system and to elucidate the adaptability mechanism of a system walking on unstable posts, the following are essential questions relating to the self-production of constraints.

(Q1) Can we explicitly describe a constraint unifying the body system and posts using the variables of the body system and post?

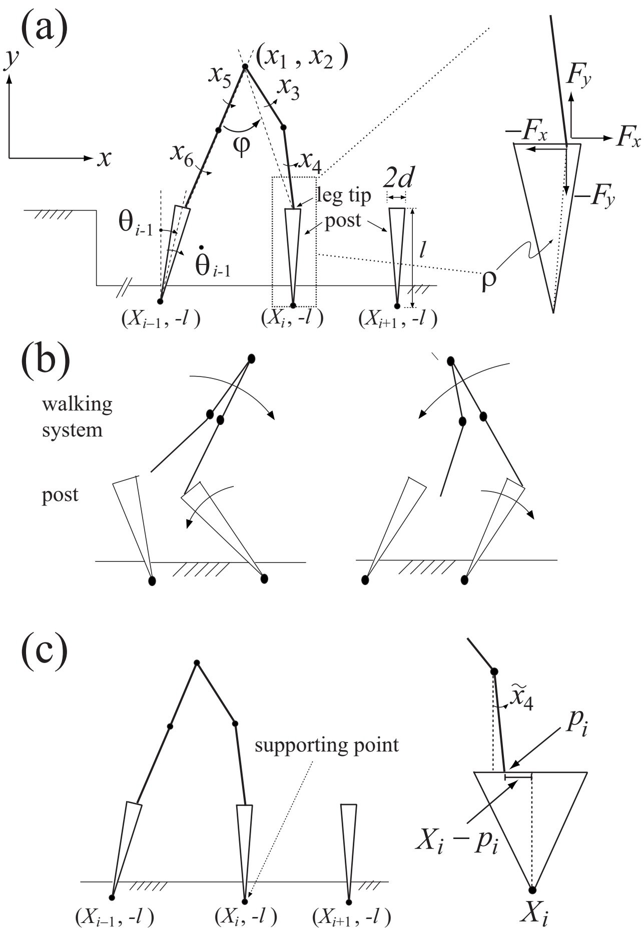

FIG. 1. Schematic of the numerical experiments. (a) The variable φ is defined as the angle between the two lines connecting the hip joint to the left tip and the hip joint to the right tip. (b) The post revolves on its bottom tip that is fixed in the ground, when the walking system places the leg tip somewhere on the post. If the walking system moves separately from the post motion, this results in failure to walk. (c) Integration of post motion into the system motion requires some constraints between them. X_i is the position of the supporting point of the i-th post, and p_i is the foot position at BSP.

(Q2) Does the system constrained by the equation found in (Q1) have a walking state basin of attraction with a similar structure to that of the system walking on flat terrain? In other words, can the system walking on unstable posts estimate the system's state using the global variables?

(Q3) Is the adaptability of the system improved by governing the initial state through global variables?

In this article, we show that the existence of two constraints, that is, the constraint unifying the body system and posts, and the constraint by global variables, enables the system to walk on unstable posts and improves the system's performance. We consider (Q1) and (Q2) in Sec. IV, and (Q3) in Sec. V.

II. MODEL

The model is composed of a walking system and posts erected in the ground. The walking system used here is the same as that described in a previous study [4], and consists of neural and body systems. The motion of the body is represented by second order differential equations of a vector $(x_1, \ldots, x_6)$ describing five links [Fig. 1(a)]: x_1 (m) and x_2 (m) represents the position of the hip joint in x axis and y axis, respectively; x_3 (rad) and x_4 (rad) [x_5 (rad) and x_6 (rad)] represent the angles of the left (right) thigh and shank with respect to the vertical, respectively. The equations are expressed according to the Newton-Euler method. The posts are modeled as rigid bodies. The weight and length of the body segments are arbitrarily set based on an adult male. The amplitude of the torques, which modulates the hip and knee joint angles, is restricted to a realistic level (see [4] and [12]).

Each post can revolve in the x-y plane about its bottom tip that is fixed into the ground. Against the revolution of a post, there is a small viscoelastic resistance. The interaction between the walking system and posts is assumed to be the following. Initially, all the posts are vertical. When the leg tip (representing the ankle joint) of the walking system contacts the upper surface of a post, sufficient viscoelastic force occurs at the contact point such that the leg tip does not slip. In this study, no active torque is produced at the leg tip, that is, $T_{r3}=T_{r6}=0$ (see the Appendix in [4] for the notation). Therefore, the walking system acts on the posts by placing the leg tip on the upper surface of the posts. By controlling the placement position, the system can govern the dynamics of the posts. The control of foot placement is executed by modulating the posture at BSP [4]. The dynamics of the angle of the i-th post relative to the vertical axis, θ_i (rad), is described by

$$I\ddot{\theta}_i = -F_y\rho \sin \psi_i + F_x\rho \cos \psi_i - b\dot{\theta}_i - k\theta_i - \frac{2}{3}mgl \sin \theta_i,$$

where a dot indicates the derivative with respect to time, $\psi_i=\psi_i(t)$ (rad) is the angle between the line connecting the tip of the leg to the rotating center of the post and the vertical axis, $\rho=\rho(t)$ (m) is distance from the bottom tip of the post to the tip of the leg (foot), F_x (N) and F_y (N) denote friction and normal force, I is the moment of inertia, b and k are the coefficients of viscosity and elasticity against revolution of the post on its supporting points, and m and g are the mass of the post and the acceleration due to gravity. Here, each of the parameter values is set to be the same for all posts: the width of the upper surface of the post d is 0.2 (m), the height of the post l is 0.5 (m), $m=3.0$ (kg), $k=100.0$ (N/m^2), and $g=9.8$ (m/s^2). The viscosity of post revolution is set to $b=70$ (N·m·s). It is assumed that there is no interaction between the posts. The position of i-th post is denoted by $(X_i, -l)$ $(i=1,2,\cdots)$.

III. BASIN STRUCTURE

First, we consider the dynamics of the system walking on flat terrain. In a previous study [4], the posture at BSP, referred to as the "initial state," is crucial for the successful generation of the following step. We find a criterion that determines whether the system proceeds to the walking state or falling state by taking an appropriate Poincaré section at the initial states. In order to do this, we need to generate various initial states, which are done by adding a small perturbation to the left leg in the swing phase. The perturbations are given by

$$F_j = F/\sqrt{(\delta F_1)^2 + (\delta F_3)^2 + (\delta F_4)^2} \times \delta F_j,$$

$$j = 1,3,4, \quad F = \{50,51, \ldots ,90\},$$

where F_1, F_3, and F_4 are external forces applied to the hip joint (x_1), the hip joint angle (x_3), and the knee joint angle (x_4), respectively. These perturbations vary the walking velocity, and the angle of the hip and knee joints at BSP. A positive F_1 increases walking velocity, while positive F_3 and F_4 decreases the hip joint angle and the knee joint angle, respectively. The details of the equations for external perturbations are described in the Appendix in [4]. The values of δF_1, δF_3, and δF_4 are selected randomly from -0.1 to 0.1 and the perturbation is applied to each joint for 0.1 s. The sample size for each F is 200, giving a total sample size of 8200.

In Fig. 2(a), a band of the stable walking regime can be observed by projecting the initial states (BSP) onto (z_1, φ) space, where $z_1 := dx_1/dt$. The variable $\varphi(t)$ is a global angle defined by

$$\varphi(t) := \varphi_l(t) - \varphi_r(t),$$

where

$$\varphi_l(t) = \tan^{-1}\left(\frac{l_1 \sin x_3(t) + l_2 \sin x_4(t)}{l_1 \cos x_3(t) + l_2 \cos x_4(t)}\right),$$

$$\varphi_r(t) = \tan^{-1}\left(\frac{l_1 \sin x_5(t) + l_2 \sin x_6(t)}{l_1 \cos x_5(t) + l_2 \cos x_6(t)}\right),$$

[see Fig. 1(a)]. The three phases are clearly separated, with little overlap between them. In other words, the basin of attraction of walking states is isolated from the falling state. This means that we can check whether the system will fall or not by checking whether the variable set (z_1, φ) is in the basin of attraction; that is, the relationship between z_1 and φ can constrain the final state of the system. When the initial states are projected on (z_1, x_3) space, such criterion is not found. When the initial states are projected onto (z_1, x_4) space, a band of the stable walking regime can be seen. However, the two regimes are not clearly separated, so the final state of the system cannot be determined by x_4 and z_1. These numerical results indicate that the type of solution is not determined by (x_3, z_1) or (x_4, z_1) at BSP [Fig. 2(c)] but that z_1 and φ are special variables for the system. We refer to z_1 and φ, which encode the basins, as global variables.

A study of bipedal walking on flat terrain [4] has shown that coordination of the global angle φ by z_1 can constrain the system into the walking state against various perturbations. The crucial role of z_1 in controlling the walking system is also seen in the present results. The constraint imposed between the dynamics of z_1 and φ is discussed in Sec. V.

IV. CONSTRAINT FOR WALKING ON POSTS

Next we consider the system walking on the posts. From a mechanical viewpoint, we expect that placing the leg tip on the post easily starts the post revolving, which leads to acceleration of the leg tip position. This acceleration makes it extremely difficult to produce the inverted pendulum motion, which is the basis for the generation of bipedal walking. Thus, walking is structurally destabilized on unstable posts. If no constraint between the dynamics of body and that of posts is given, the system cannot maintain walking on posts. In fact, when X_i ($i = 1,2, \ldots,$) are given randomly satisfying $|X_i - p_i| < 0.01$ at every step [see Fig. 1(c)], the system falls down within about 10 steps.

We show that the system can control angular velocity and acceleration of the post by controlling foot position on the

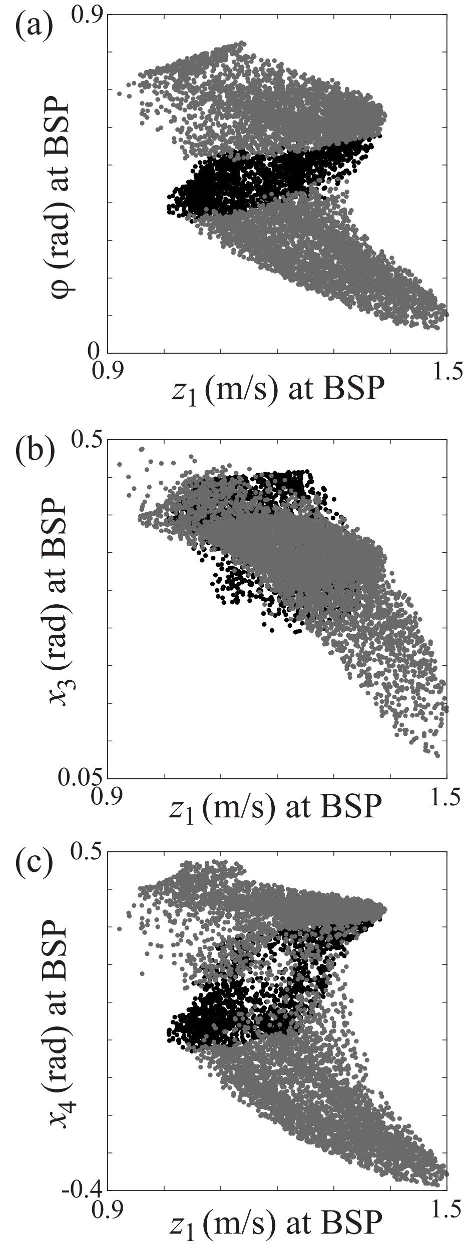

FIG. 2. Response of the system to perturbations when the system walks on flat terrain. Black and gray points indicate the initial data for which the system finally walks and fails to walk, respectively. The initial states are projected onto z_1-φ space [(a)], z_1-x_3 space [(b)], and z_1-x_4 space [(c)].

foothold depending on the body variables, and such control enables the system to maintain walking. That is, stable walking is established by constraining the system variable (or the position of foothold) by system variables. In addition, we show that a basin structure similar to that shown in Fig. 2(a) appears by restricting the solution orbit to an appropriate subspace. Such restriction corresponds to regarding the walking system and the post system as a unified system.

Walking on unstable posts is stabilized by unifying the dynamics of the body and the posts. Let us consider a situation in which the system walks to the right and the left leg is in the swing phase. The body system falls forward faster as the knee joint angle of the left leg at the BSP decreases because the horizontal position of the center of mass of the left leg decreases as the knee joint angle decreases. In order to stabilize the walking on the posts, the post should revolve in accordance with the walking speed; the revolving speed should be increased (decreased) as the walking speed is increased (decreased). Since the position of the center of mass of the left leg at the BSP heavily depends on $\sin x_4$ and the revolving speed of the post is controlled by the foot position of the left leg at BSP, p_i, we assume a constraint between $\sin x_4$ and $X_i - p_i$. The same argument is valid for the right leg. Thus, we assume the constraint

$$X_i - p_i = \alpha \sin \tilde{x}_k, \qquad (1)$$

where $k=4$ ($k=6$) for the left (right) leg, and $X_i - p_i$ is distance of the foot position from the central point of the surface of the post. The variables $\tilde{x}_4$ and $\tilde{x}_6$ are x_4 and x_6 at BSP, respectively [Fig. 1(c)], and $\alpha \in \mathbb{R}$ is the control parameter. According to Eq. (1) the system needs to set the angle of the hip and knee joints during the swing phase such that the values of $\tilde{x}_4$ ($\tilde{x}_6$) and p_i at every BSP satisfy Eq. (1). However, since such control is quite difficult in numerical experiments, we employ an alternative setting in which the position of a post X_i ($i=1,2,\ldots,$) is not determined at $t=0$ but at each BSP. That is, X_i is given depending on $\tilde{x}_4$ ($\tilde{x}_6$) and p_i at the moment the height of the leg tip from the surface becomes 0. The advantage of this experimental setting is that we can precisely specify $X_i - p_i$ satisfying (1) in numerical simulations. We refer to the constraint in Eq. (1) as "constraint I."

In order to determine the effectiveness of constraint I numerically, the following experimental procedure was used. First, the system walks on flat terrain until step number I_s, which is taken to be large enough that the walking behavior converges to the limit cycle. Next, the system walks on the posts from step (I_s+1) to step I_e. The system then walks again on flat terrain from step (I_e+1). Here, we fix $I_s=13$ and $I_e - I_s = 50$.

The experimental results show that the system behavior depends critically on α. Figure 3 shows the step number that the system walking on posts achieves for various values of α. For $\alpha \leq 0$ and $\alpha \geq 0.5$, the system falls within 10 steps. This indicates that constraint I is crucial to maintain walking on posts.

In order to investigate basin structure of the unified system, we execute numerical experiments (similar to those conducted for flat terrain) to find the basin structure in the subspace defined by Eq. (1). Small perturbations F_1, F_3, and

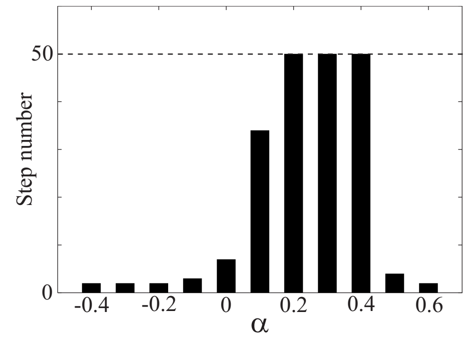

FIG. 3. Step number that the system walking on posts achieves for given α. The maximum step number is 50 since $I_e - I_s = 50$.

F_4 are applied during the swing phase between steps I_s and (I_s+1). As for walking on flat terrain, we take δF_1, $\delta F_4 \in [-0.1, 0.1]$ but now we take $\delta F_3 \in [-0.01, 0.01]$. An isolated band of walking states clearly appears when initial states are projected onto (z_1, φ) space [Fig. 4(a)].

It should be noted that the isolated basin for the system walking on posts is found by reducing the range of δF_3 from $[-0.1, 0.1]$ to $[-0.01, 0.01]$. When the range of δF_3 is taken to be same as that for the case of flat terrain, a lot of falling states are interspersed in the band pattern of walking states [Fig. 4(b)]. This indicates that the basin becomes thinner along x_3 axis. Therefore, strong variability of the hip joint

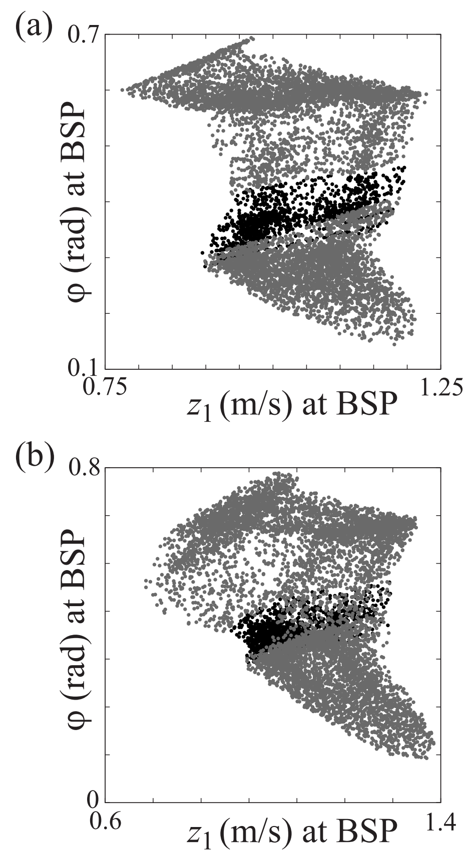

FIG. 4. Response of the system to perturbations when the system walks on the posts for (a) $\delta F_1, \delta F_4 \in [-0.1, 0.1]$ and $\delta F_3 \in [-0.01, 0.01]$ and (b) $\delta F_1, \delta F_3, \delta F_4 \in [-0.1, 0.1]$. An isolated band of the walking state can be seen in (a).

angle during the swing phase induces falling. This finding is useful for constructing the adaptability function discussed in Sec. V.

This series of numerical experiments shows that constraint I enables the unified system to establish inverted pendulum motion and generate walking. That is, in phase space, the solution orbit of the system can be enclosed in the attractor basin of the walking state by constraint I. In addition, it was found that the unified system can estimate its state using the global variables z_1 and φ.

V. CONSTRAINT FOR EXTERNAL PERTURBATIONS

In real life cases of the integration of environmental objects into locomotion, such as walking on stilts, the integrated locomotion system adapts to external perturbations. In order to reproduce such adaptability, we improve the unified system induced by constraint I and show that a constraint between global variables expands the basin of stable walking.

Control of the leg tip position on a post, or posture formation at BSP, requires modulation of the joint angle of the hip or knee during the swing phase. Since modulation of the hip joint angle during walking on the posts easily causes falling, as described above, the hip joint torque should not be greatly varied when the system is affected by external perturbations. Therefore, we assume that when the system is affected by external perturbations, the system modulates posture by altering the knee joint angles. Thus, φ at BSP is controlled by modulating the knee joint angles

$$\phi_k(t) = \bar{\phi}_k + \alpha_f g[z_1(t) - \bar{z}_1] + \alpha_b g[\bar{z}_1 - z_1(t)], \qquad (2)$$

where $g(z)=z$ for $z>0$ and $g(z)=0$ otherwise, $\bar{\phi}_k$ is a basal angle and is set to 0.13π (rad), $\bar{z}_1=0.95$ (m/s) is set close to the average speed, $\alpha_f=-0.3$, and $\alpha_b=0.2$. The function in Eq. (2) is constructed so that the posture at BSP can approach the band pattern shown in Fig. 4(a); when $z_1>\bar{z}_1$, the global angle φ for the leg in the swing phase is reduced through Eq. (2) and vice versa for $z_1<\bar{z}_1$. We refer to Eq. (2) as "constraint II."

In order to test the effectiveness of constraint II, a horizontal external perturbation F_1 is applied to the hip position from the 15th step. This may resemble the situation when a walking human is buffeted by wind. The system including Eq. (2) can maintain walking for at least 50 steps for F_1 from −14 (N) to 13 (N), while the system without Eq. (2) can adapt only for F_1 from −9 (N) to 4 (N). These results show that constraint II improves the system's performance in the presence of perturbations. Theoretically, constraint II enables the solution of the system to be returned to the attractor basin in the restricted subspace.

Finally, we remark that the system with constraint II but without constraint I cannot maintain walking on posts. In fact, when the position of a post X_i is given randomly satisfying $|X_i-p_i|<0.01$ ignoring constraint I, the system maintains walking for less than 10 steps on average. This indicates that constraint I is indispensable for maintaining walking on posts. In addition the greater adaptability against external perturbations occurs only when constraint I is used. In other words, the walking system can adapt to external perturbations by using multiple constraints: constraint I should be complemented by constraint II.

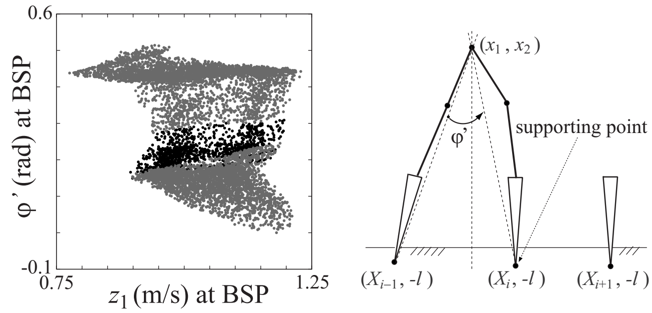

FIG. 5. Response of the system to perturbations when the system walks on the posts for $\delta F_1, \delta F_4 \in [-0.1, 0.1]$ and $\delta F_3 \in [-0.01, 0.01]$. Black and gray points indicate the initial data for which the system finally walks and fails to walk, respectively. The initial states are projected onto z_1-φ' space where the angle φ' is defined by the position of the vertex and the boundary of the whole system.

VI. DISCUSSION

We have described a coordinate system that consistently reveals the basin structure of a walking system before and after importation of environmental objects. The future of the walking system is constrained by the coordinate variable z_1 and global angle φ, even if environmental objects are imported. It was found that two constraints are necessary to establish walking on environmental objects in the presence of perturbations. (i) Constraint I encloses the solution orbit of the system in the attractor basin of the walking state; this corresponds to a unification of walking system behavior and environmental object motion. (ii) Constraint II expands the basin, which returns the solution previously shifted outside by perturbations back to the basin; this corresponds to the unified system's adaptation to external perturbations. Thus, for the integration of the environmental objects into the system, self-production of constraints is attained by two hierarchical processes.

One interesting observation is that the band pattern also appears by projecting the initial states onto z_1-φ' space instead of z_1-φ space, where φ' is defined by the position of the vertex and the boundary of the whole system: $\varphi'(t) = \tan^{-1}[x_1(t)-X_i/x_2(t)+l] - \tan^{-1}[x_1(t)-X_{i-1}/x_2(t)+l]$ (Fig. 5). This also indicates that the whole system including the posts can be regarded as a bipedal locomotion system that has a basin structure similar to the system walking on flat terrain. This also implies that, if the body system can sense the position of the tip of post as if the post were a segment of the body system, the whole system can control its behavior similar to the way that the system walking on flat terrain does. We also remark that the overlap between the walking region and the falling region in z_1-φ' space seems to be less than that in z_1-φ space, which indicates that φ' may be more suitable than φ as a global variable that determines the system's convergent state. It is important, from a descriptive viewpoint, that the constraints proposed here are given in terms of variables expressing the global state of the whole system: z_1 is the velocity of the whole system, and φ' is defined by the position of the vertex and the boundary of the whole system. We expect that the self-production of constraints, established by variables expressing the global state of the whole system, is a key mechanism in embodiment phenomena.

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